8.3 Geometric Sequences and Series

Definition of Geometric Sequence

A sequence is geometric if the ratios of consecutive terms are the same. So, the sequence $a_1$, $a_2$, $a_3$, …., $a_n$,…. is geometric if there is a number r, such that $a_2/a_1 = a_3/a_2 = a_4/a_3 = \dots$ r. R≠0.

The number r is the <u>common ratio</u>* of the sequence. *kind of like common difference but for x/÷

Ex. a) 2,4,8,16,...,$2^n$,... r=2

b) 12, 36, 108, 324,...,$4(3^n)$,... r=3

The $n^{th}$ term of a geometric sequence has the form

$a_n = a_1r^{n-1}$

where r is the common ratio of consecutive terms of the sequence

$a_1, a_1r, a_1r^2, a_1r^3, a_1r^4, ..., a_1r^{n-1},...$

Ex. Write the first five terms of a geometric sequence whose first term is $a_1$=3 and whose common ratio is r=2.

$a_1 = 3(2^0) = 3$

$a_2 = 3(2^1) = 6$

$a_3 = 3(2^2) = 12$

$a_4 = 3(2^3) = 24$

$a_5 = 3(2^4) = 48$

The Sum of a Finite Geometric Sequence

r≠1 $S_n = \Sigma^n_{i=1}a_1r^{i-1}=a_i(1-r^n/1-r)$

Ex. Find the sum $\Sigma^{12}_{i=1}4(0.3^n)$*

$= 4(0.3)\ [1-(0.3^{12})/1-0.3] = 1.714$

*If the summation started at 0, then you would $a_1+a_1(1-r^n/1-r)$